Chapter 6

Variable Smoothing Lengths and High Resolution

The density structure within the ballistic stream for the initial model was discussed in section 4.2.2. Due to the limited resolution of the initial models, the detailed density structure was uncertain. A higher resolution study was needed to determine if any asymmetries existed in the density profile. In addition a higher resolution study serves to confirm that the initial model results did not suffer from inaccuracies that might occur if the resolution was not sufficient to resolve the significant physical processes.
6.1 Variable Smoothing Lengths

As discussed in section 3.2.2, the smoothing length determines the resolution of the SPH model, and SPH can be implemented with either fixed or variable smoothing lengths.

The initial reference model used a fixed smoothing length implementation since this is computationally simpler to setup. However, this is not the most efficient method since the stream density varies over a considerable range. In particular, the high density regions of the stream are significantly over sampled, while the lowest density regions are borderline under-sampled. Figure 6.1 shows the number of neighboring particles which contribute to the density at each SPH particle position plotted verses the x position of each particle. The very beginning of the stream near L1 has such a high density that particles had up to 3300 neighbors. The computational efficiency is very low in this region which also contains the majority of the particles. The rest of the stream, as seen in the close-up portion of Figure 6.1, contains particles which have less than 50 neighboring particles. A variable smoothing length model would be much more efficient.

A variable smoothing length version of the initial reference model, hereafter referred to as the variable reference model, was developed. The variable
Figure 6.1: The number of neighboring particles for the initial reference model with constant smoothing length, showing the full range (top) and a closeup (bottom).
reference model had all of the same system parameters as the initial reference model described in Chapter 4. The number of particles in the model stayed at N=20,000, but the smoothing length was allowed to change depending on the density.

In variable smoothing length implementations of SPH the smoothing length is related to the density by the general equation 3.13 discussed in Chapter 3. A value of \( k = 1.25 \) was determined to provide a reasonable number of neighboring particles for the majority of the stream, yielding the equation:

\[
h_i = 1.25 \left( \frac{\rho_i}{m_i} \right)^{-\frac{1}{3}}
\]  

(6.1)

where the density determined at the previous time step is used to determine the smoothing length for the next time step. A maximum smoothing length of \( h = 0.003 \) was imposed on the variable reference model to maintain the minimum resolution of the initial reference model.

Figures 6.2 shows the smoothing lengths as a function of x position for each particle in the variable reference model. The smoothing length for particles near L1 are much lower than the maximum smoothing length, improving
the resolution in that region. Most particles downstream of $x = 0.20$ still have $h = 0.003$. The average smoothing length for all of the SPH particles is $h = 0.0019$.

![Smoothing lengths for the variable resolution model.](image)

Figure 6.2: Smoothing lengths for the variable resolution model.

Figure 6.3 shows the number of neighboring particles for the variable reference model plotted verses the $x$ position for each particle. The average number of neighboring particles over the whole stream is 51.4 neighbors per particle. This is a good number, well above the recommendation of at least 20 neighbors and still computationally efficient. The number of neighboring particles does drop off as the density of the stream decreases for lower $x$
values, but even in the region $x < 0.20$ the average number of neighbors is 29.8 per particle. Only 8% of the stream particle had less than 20 neighbors, and most of those were on the edges of the stream or in the lowest density region of the stream beyond where the stream should couple to the field lines.

![Graph](image.png)

Figure 6.3: The number of neighboring particles for the variable resolution model.

The ballistic streams for the initial and variable reference models are shown in Figure 6.4. There are slight differences in the beginning part of the stream. The initial reference model shows a narrowing of the stream at the very start and the variable reference model does not. This is probably due to a slight over smoothing of the pressure gradients in this region where the
thermal pressure is most important. The remainder of the streams are very similar to each other.

The coupling regions for the two models are also very similar with only slight differences. Table 6.1 lists the main coupling region data while Figure 6.5 shows the coupling region plots. The largest difference between the two models is the amount of material coupling near L1 for the variable reference model. Because of the increased resolution near the L1 point, the overall gaussian distribution is less smoothed out and the particles at the edges will have a lower density. As discussed already, however, the coupling of material near L1 is suspect. The shift in the average coupling position in the main coupling region is related to the increased coupling near L1. By moving particles which had coupled at the start of the main region to the L1 coupling region, the average coupling position moves outward toward the white dwarf. The end of the coupling region also shifts slightly towards the white dwarf. The slightly higher resolution in the center of the stream has increased the core density just enough to penetrate further into the magnetosphere. A difference of only 0.4 $R_{wd}$ however is unlikely to be observable.
<table>
<thead>
<tr>
<th>Model</th>
<th>average distance from WD ($R_{wd}$)</th>
<th>start of coupling region ($R_{wd}$)</th>
<th>end of coupling region ($R_{wd}$)</th>
<th>extension in x direction ($R_{wd}$)</th>
<th>percentage of coupling in main region</th>
</tr>
</thead>
<tbody>
<tr>
<td>initial reference</td>
<td>18.7</td>
<td>24.2</td>
<td>15.4</td>
<td>10.9</td>
<td>93.6%</td>
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<tr>
<td>variable reference</td>
<td>17.9</td>
<td>24.2</td>
<td>15.0</td>
<td>11.5</td>
<td>89.3%</td>
</tr>
</tbody>
</table>

Table 6.1: Data for the main coupling regions for the initial reference model and the comparison model using variable smoothing lengths.
Figure 6.5: Coupling positions for the initial reference model (top) and the variable reference model (bottom).
6.2 High Resolution Model

The initial reference model, described in Chapter 4, had a resolution roughly one fifth of the stream width. A resolution of one tenth the stream width, $h = 0.0015$, would be much better to explore the stream structure. Once variable smoothing lengths are used, this resolution level becomes practical, although the computation time becomes much greater. To handle the much longer computational times, my advisor Steve Howell and myself applied for and received a small allocation of supercomputer time from the National Center for Supercomputing Applications at the University of Illinois Urbana/Champaign.

The high resolution reference model was created using the same system parameters as the initial reference model and keeping the same model assumptions as described in Chapter 4. The number of particles in the model was increased from $N=20,000$ to $N=500,000$. The average smoothing length over the entire stream was $h = 0.0008$ while the average smoothing length for just the particles downstream from $x=0.20$ was $h = 0.0014$, indicating the resolution is slightly better than the goal. The smoothing lengths of each particle plotted verses x position in the stream are shown in Figure 6.6.

The ballistic streams for the variable reference model and the high reso-
Figure 6.6: Smoothing lengths for the high resolution model.

The streams appear virtually identical to each other, with the obvious exception that the high resolution has more particles plotted than the variable resolution model. Neither of the streams show the narrowing of the stream near L1 that was seen in the initial reference model.

The density structure within the stream can be studied in more detail by extracting the density at various cross-sections along the stream as was done for the initial reference model in Section 4.2.2. To improve the accuracy of the density measurements, extraction were taken for five times after steady
Figure 6.7: Ballistic stream for the variable reference model (top) and the high resolution reference model (bottom).
state had been reached. The average of these five times provides a better estimate of the general stream structure since random deviations present in the individual extractions will be suppressed. Each individual extraction was also examined to determine if they contained any significant deviations from the average above the low level of random errors, and they did not.

The FWHM measurements for both the initial reference model and the average high resolution model are given in Table 6.2. The high resolution model shows that the initial reference model overestimated the stream width, an effect related to the larger smoothing length in the initial model. The high resolution model shows a fairly constant stream width from the L1 point to the center of mass at which point it begins to narrow.

After looking at the stream widths, the density distributions were examined for asymmetry. No significant asymmetry was detected in the x=0.40 extraction in the high resolution model, as would be expected since material starts out from the L1 point with a symmetric distribution. By the time the stream has reached x=0.35 just downstream from the L1 point, however, asymmetries start appearing particularly in the wings of the distribution. Figure 6.8 shows one of the averaged density distribution, at x=0.20, plotted as a solid line. A mirror image of the density distribution is over-plotted
<table>
<thead>
<tr>
<th>x position</th>
<th>initial model FWHM</th>
<th>high resolution FWHM</th>
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</thead>
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<tr>
<td>0.40</td>
<td>0.0078</td>
<td>0.0088</td>
</tr>
<tr>
<td>0.35</td>
<td>0.0111</td>
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<td>0.25</td>
<td>0.0132</td>
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<tr>
<td>-0.05</td>
<td>0.0120</td>
<td>0.0076</td>
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<tr>
<td>-0.10</td>
<td>0.0126</td>
<td>0.0064</td>
</tr>
</tbody>
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Table 6.2: Streams widths for the initial and high resolution reference models at each cross-section position, in dimensionless units.

as a dotted line to accentuate the asymmetries. In general the core of the stream is slightly offset to the outside of the stream (y > 0) while the wings are offset towards the inside edge of the stream (y < 0). This small asymmetry could easily be dismissed as some effect of random errors, however, the general trend is present in all of the individual exactions over a long range of the stream from x=0.35 to x=0.00. Measuring the width of the stream at the point where the density drops to zero, the asymmetry becomes more dramatic with a maximum at x=0.35 where the inner edge extends 1.2 $R_{wd}$ further out from the stream center than the outside edge of the stream.

These asymmetries follow the general patterns seen in the non-interacting
Figure 6.8: Density profile for the high resolution stream at the position x=0.20 (solid line) shown with a mirror image of the density profile (dotted line).

models of Chapter 2 but are much less dramatic, indicating that the same gradient in the forces across the stream are responsible for them but the thermal pressure forces dampen the effect. As interesting as these asymmetries may be from a theoretical standpoint, however, the general density distribution can be qualified as symmetric. The low levels of asymmetry which are present are unlikely to affect any observable attributes.

Examining the coupling region for the high resolution model shows the
same features seen in the initial reference model. Figure 6.9 shows the coupling positions for the high resolution reference model and the variable reference model for 300 time-steps after steady state has been reached. The greater number of points in the high resolution model hides the details within the coupling region, but the major features are similar to the variable reference model.

The most visually obvious difference would be the material coupling along the edges of the stream extending much further towards the L1 point that the variable reference model shows. In actuality, the particles which couple between x=0.10 and x=0.35 account for only 1% of the total, and do not represent a significant difference between the streams. The vast majority of the particles still couple in the main coupling region which is in the same basic location as in the initial and variable reference models. Table 6.3 lists the relevant data. The average particle position within the high resolution model’s main coupling region is slightly closer to the white dwarf than the variable reference model, indicating that the denser stream core could penetrate further into the magnetosphere than was predicted for the somewhat over smoothed initial reference model. The difference is still much smaller than the shifts seen for different system parameters seen in Chapter 5.
<table>
<thead>
<tr>
<th>Assumption Used</th>
<th>average distance from WD ($R_{wd}$)</th>
<th>end of coupling region ($R_{wd}$)</th>
<th>extension in x direction ($R_{wd}$)</th>
<th>percentage of coupling in main region</th>
</tr>
</thead>
<tbody>
<tr>
<td>variable reference</td>
<td>17.9</td>
<td>15.0</td>
<td>10.9</td>
<td>89.3%</td>
</tr>
<tr>
<td>high resolution variable</td>
<td>17.4</td>
<td>14.2</td>
<td>27.2</td>
<td>83.1%</td>
</tr>
</tbody>
</table>

Table 6.3: Data for the main coupling regions for the variable reference model and the comparison model using high resolution variable smoothing lengths.

Figure 6.10 shows contours of the high resolution coupling region, revealing the familiar underlying structure. Just as in the initial reference model coupling region explored in section 4.2.3, the coupling occurs first off of the outside edges of the stream with the high density core coupling last. The asymmetry of the inside edge of the stream coupling before the outside edge is still evident.
Figure 6.9: Coupling positions for the variable reference model (top) and the high resolution model (bottom).
Figure 6.10: Contours of the main coupling region in the x-y plane, showing the levels for 10, 50, 100, 150, 200, and 250 particles coupling per bin. The dotted line shows the ballistic trajectory for comparison.
6.3 Summary

The high resolution model was run to answer two major questions. First, was is the detailed structure of the ballistic stream and is it asymmetric? Second, does an increase in resolution result in any significant changes in the accretion stream geometry? These questions have now been answered.

The high resolution models confirmed that the ballistic stream is a narrow region with the peak density falling along the path of the base trajectory. The distribution around the base trajectory is largely symmetric but has a small systematic asymmetry due to the greater pull of gravity on the inner edge of the stream compared to the outer edge. This small asymmetry is at a low enough level, however, that a symmetric stream is a reasonable approximation.

The general features of the accretion stream geometry have not changed in the high resolution model, indicating that the lower resolution models were adequately resolved for a general investigation of the stream. The most noticeable changes between the high resolution model and the initial reference model is the width of the ballistic stream and the slight shifting of the coupling region. Overall the differences would not significantly change the appearance of the stream except perhaps in the highest quality observations,
for which the higher resolution models would be more appropriate.