Other Dark Matter and Dark Energy Universes

Introduction

One of Einstein's most important equations enables physicists and astronomers to understand how the contents of our universe determine what kind of universe we observe. You will study a version of that equation in this lab, and you will discover what universes are possible other than the one where we live. The Einstein equation that relates to the universe is a consequence of his General Theory of Relativity and it requires a high level of mathematical sophistication to understand. Luckily, there is an easier way to understand the results of the theory at a much lower level of math by using a fundamental principle of physics, *energy conservation*: energy cannot be created or destroyed but only converted from one form to another.

We know that the universe contains matter, and we also know that this matter is often in motion. But what prevents a moving ball, a ball that has been tossed upwards, for example, from escaping into space? Gravity, of course, pulls the ball back down to earth before it can escape. The energy of motion is called *kinetic energy* and it is always *positive*. The gravity trap is called *gravitational potential energy*; it is always *negative* and it depends *inversely* on the ball's distance from the earth's center. These two forms of energy combine to give constant total energy so that the ball's energy is conserved:

(kinetic energy of ball) + (gravitational potential energy of ball) = (total energy)   (1)

Let's consider three possible values for the ball's total energy that will correspond to the three possible values for the total energy or any universe. The total energy can be negative. That means that the negative gravitational potential energy (GPE) dominates the positive kinetic energy (KE) in equation (1), and the ball remains trapped near the earth's surface. As the ball moves upwards, we know from experience that it slows down, meaning that the positive KE approaches zero. This equation tells us that the negative GPE must also approach zero because the total energy is constant—a negative constant.

Question 1. What is the ball's kinetic energy at the instant that it stops, before falling back to earth?

Before falling, the ball stops moving, so its KE (the energy of motion) is zero. When the KE is zero, we recognize this as the instant that the ball reaches its maximum height and, therefore, when the ball's GPE is at a maximum (still negative).

The ball's total energy can be positive. That means that the positive KE dominates the negative GPE in equation (1), and the ball escapes the earth's surface.

Question 2. Does the ball (a) slow down, (b) speed up, or (c) keep the same speed as it moves away from the earth?

The ball slows down as it moves away from the earth because the negative GPE approaches zero. Because the total energy is a positive constant, the KE must decrease by the same amount as the negative GPE is reduced.

And the ball's total energy can be zero. The KE balances the GPE exactly in equation (1).

Question 3. What happens to a ball thrown upwards in this case (ignoring air resistance)?

A ball thrown upwards in the case where the total energy is zero will move away from the earth's surface with an ever decreasing speed. The negative GPE approaches zero as the ball moves away, therefore the KE must also approach zero so that the total energy remains at zero.
In this lab we will investigate universes with negative, positive, and zero total energy.

To explore the various possible universes, we must convert the above general statement (1) of energy conservation into a mathematical equation. First, however, we need to define all of the quantities that will appear in the equation. According to the Big Bang theory of how the universe began and evolves, the entire universe began expanding immediately after it came into existence. That means that any two points that were initially some distance apart now have that distance between them stretched by a factor s, so let's call s the size factor. The size factor, s (or scale factor, a, as it's called by astronomers), tells us the size of the universe relative to its size today: $s = 1$ means the size of the universe today, $s < 1$ means a smaller universe than today's, and $s > 1$ means a larger one.

s = size factor (or scale factor) of the universe relative to its size today   (4)
v = rate of size factor growth or shrinkage (speed of universe growth or shrinkage)

We want to know how the size factor changes as the universe ages, and that depends on all of the types of matter and energy in the universe. So far, physicists and astronomers have discovered that microwave radiation (from the hot Big Bang, called Cosmic Microwave Background or CMB), ordinary matter (mostly protons, neutrons, electrons), dark matter (source unknown), and dark energy (source unknown) make up all of the types of matter and energy in the universe. Although the sources for dark matter and dark energy are unknown, they do have observable gravitational effects. Just as the earth's gravity determines whether the ball leaves or returns to earth, the various types of matter and energy determine whether the universe continues to expand, or begins at some time to contract.

We will measure the different types of mass and energy as a fraction of the energy density needed to make the universe "flat," meaning a universe where two parallel lines never meet. Here are the symbols for the fractions of mass and energy that we'll use.

f_{rad} = fraction of radiation energy   (5)
f_{matter} = fraction of matter mass, ordinary and dark
f_{de} = fraction of dark energy
$f_0 = f_{\text{rad}} + f_{\text{matter}} + f_{\text{de}}$ is the total fraction of mass and energy
$f_0 - 1$ is curvature: positive gives a closed universe, zero gives a flat universe, and negative gives
        an open universe

Note that in exploring the different possible universes, any of the fractions can be made greater than 1.

We are ready to see what Einstein's equation tells us about the expansion (or contraction) of the universe. The equation is actually named after Friedman-Robertson and Walker (FRW) because they figured out how to make Einstein's equation practical for studying universes:

$$\frac{v^2}{H_0^2} = \frac{f_{rad}}{s^2} + \frac{f_{matter}}{s} + f_{de}s^2 - (f_0 - 1).$$ (6)

We'll call this the FRW equation. Recall that an easy way to understand this equation is to view it as energy conservation, equation (1) above, so that it's not as complicated as it looks. Cosmologists (scientists who study the origin, contents, and evolution of the universe) use calculus to solve this equation, but we'll solve it by using a spreadsheet. First, we must determine the meaning and value of the quantity H_0, called the *Hubble constant*.

Outline of Activities

- **Hubble Constant**
  Goals: – Plot the speed of universe's expansion versus the distance
                     – Determine the Hubble constant from the plot

- **Empty Universe with Negative Curvature**
  Goals: – Plot the size factor (scale factor) for the empty universe
                     – Determine the age of the empty universe

- **Matter-Dominated Universe with Positive Curvature**
  Goals: – Plot the size factor for the matter-filled universe
                     – Determine the relative size of the universe at different times after the
                        Big Bang

- **Dark Energy-Dominated, Flat Universe**
  Goals: –Plot the size factor for the universe with and without dark energy
                     –Determine the age of the universe

Hubble Constant

       In 1929, Edwin Hubble discovered that the universe is expanding by observing that galaxies are moving away from us. He was able to use the brightness of certain stars (Cepheids) in the receding galaxies to determine their distance from us. But if some galaxies are farther away from us than others, how did they get to be so much farther away if the near and far galaxies travel for the same amount of time? Obviously the more distant galaxies are moving faster than closer ones. This is represented by a simple formula called the *Hubble Law*:
$$v = H_0 r , \qquad (7)$$
where v is the speed of the galaxy, H_0 is the Hubble constant, and r is the distance from earth observers. The formula can be rewritten so that it looks like a formula that almost everyone has seen before:
$$r = v H_0^{-1} , \qquad (8)$$
which is analogous to the equation (distance) = (speed)(time). Making a plot of galaxy distance versus speed, therefore, should be a straight line and the slope of that line will allow us to find H_0^{-1} then H_0, needed for the FRW equation (6). We will use Hubble Space Telescope data collected in 2001 by a group of astronomers, led by Wendy Freedman, who determined a value of H_0 that was, for the first time, accurate to 10%.

Directions

1. Load the Hubble constant data into a spreadsheet.
2. Select columns A and B, then make a scatter plot.
3. Label the x-axis "Distance in Mpc" (Mpc = megaparsec), and label the y-axis "velocity in km/s."
4. Choose Trendline→Linear Trendline. You should get a straight line through the data that is the line that best fits the data.
5. Under the Trendline menu, select Trendline Options and click on "Display Equation on Chart," and "Set Intercept = 0."
6. The slope of the line is H_0, and is given by the equation in units of (km/s)/Mpc. Record your slope value: $H_0 = $ _____ .
7. Convert your value of H_0 from units of (km/s)/Mpc to units of $\frac{1}{seconds}$ by using 1 km = 10^3 m, and 1 Mpc = 3.0857×10^{22} m. $H_0 = $ _____1/sec
8. Find H_0^{-1}. $H_0^{-1} = $ _____ seconds.
   Convert seconds to years: 1 year = 3.15×10^7 seconds
   $$H_0^{-1} = \underline{\hspace{4cm}} \text{ years.}$$

In the next section and remaining sections of this lab, you will use a value near this last value of H_0^{-1}. The graph and equation (8) tell us that some galaxies have traveled a distance of only 5 Mpc during a time H_0^{-1}, while others have traveled a distance of 22 Mpc during the same time.

Question 4. What is your interpretation of the time H_0^{-1}? What is the meaning of this time? (Recall that Hubble's law has the same form distance = speed*time.)

Empty Universe with Negative Curvature

As a first exercise in using the FRW equation (6), let's explore a completely empty universe, meaning f_{rad}, f_{matter}, f_0, and $f_{de} = 0$. Notice that $f_0 - 1 = -1$, means that the universe is open: the universe expands without limit (two lines in space, initially parallel, diverge). You might think that this is completely pointless because the actual universe where we live is not empty; but you have a surprise in store. The FRW equation (6) becomes

$$\frac{v^2}{H_0^2} = 1 . \tag{9}$$

that is simplified algebraically to

$$v^2 = H_0^2 . \tag{10}$$

Although this equation is simple algebraically, the calculus is hidden because v is actually the *change* in the size factor of the universe per unit time. We wish to know the size factor (scale factor) of the universe at any time after the Big Bang. In particular, we want to know the following: at what time does an empty universe reach its present day size factor, $s = 1$? Before we begin the analysis, let's predict how this universe will behave based on the fact that all of the terms are positive in equation (9).

Question 5. When compared to equation (1) and Question 2 for the ball with positive total energy, how do you expect the universe to behave?

Directions

Entering formulas into cells by copy-and-paste is much easier than typing them, but when pasting in the spreadsheet choose Edit->Paste Special->Text
1. Open a spreadsheet, and label cell A1 "size factor (s)." It turns out to be easier to make a list of the size factors, then to calculate the corresponding times.
2. Type "0.01" in cell A2, then in cell A3 type "=A2+0.01" and drag all the way down to cell A111, where the value should be 1.1.
3. Label cell B1 "size factor change rate," then type "1" in cell B2 and copy down to cell B111 by double-clicking on the cross in the lower right corner of the cell. (The "1" is the square root of the right side of equation (7): $\sqrt{1} = 1$)
4. Your value of H_0^{-1} will be needed in the next step. Although you found a value near 13 billion years, a more careful analysis yields 14 billion years, so this is the value used.
5. Label cell C1 "yrs after Big Bang," then in cell C3 (*not* C2) type "=14*0.01*(B2+B3)/2". In cell C4 insert "=C3+14*0.01*(B3+B4)/2" and copy down to cell C111 (double-click on the cross in the lower right corner) where the last value should be 15.26.
6. To match the size factor values with the years column, type "av. size factor" in cell D1 and type "=0.5*(A2+A3)" in column D3, then copy down to D111 (double-click in the lower right corner) where the last value should be 1.095.
7. Select columns C and D, then make a smooth lined scatter plot. Label the horizontal axis as "Years after Big Bang (billions)," and the vertical axis as "size factor." Does the growth of this universe agree with your prediction in Question 5 above?
   _____
8. Recall that we want to know at what time (years after the Big Bang) does an empty universe reach its present day size factor, $s = 1$? Use the graph to answer this question.
   _____ billion years

You will be surprised to learn that your value is only slightly more than the actual age of the universe (13.7 billion years). In other words, the theory tells us that if we want to get a very good approximation for the age of the universe, we just need to assume that the universe is empty! The theory must be wrong!? But let's not jump to conclusions based on one number. Scientific validation or invalidation of a theory requires the collection of data over the entire range of the theory's prediction. This means collecting data at 2, 4, 6, etc., up to almost 14 billion years after the Big Bang, and that means receiving and analyzing light today that was emitted billons of years ago, when the universe was smaller. Furthermore, as you will see, the theory, with different fractions of matter and energy, can give nearly the same value for the age of the universe. But first, let's look at another kind of universe, one that does not simply grow in size as it ages.

Matter-Dominated Universe with Positive Curvature

Consider a universe with $f_{rad} = 0, f_{matter} = 3.1, f_0 = 3.1$, and $f_{de} = 0$ in the FRW equation (6). Notice that $f_0 - 1 = 2.1$ means that the universe is closed–has positive curvature–and this activity demonstrates what "closed" means. The FRW equation becomes

$$\frac{v^2}{H_0^2} = \frac{3.1}{s} - 2.1.$$

(11)

Notice that the left side of the equation is always positive, therefore the right side of the equation must also remain positive.

Question 6. What is the largest allowable value for the size factor, s, so that the right side of the FRW equation remains positive?

$$s < \underline{\hspace{2cm}}$$

Again, let's predict how this universe will behave. First rewrite equation (11) so that it is in the energy conservation form of equation (1):

$$\frac{v^2}{H_0^2} - \frac{3.1}{s} = -2.1.$$

(12)

Question 7. When compared to equation (1) and Question 1 for the ball with negative total energy, how do you expect this universe to behave?

Directions

Remember to choose Edit->Paste Special->Text when pasting formulas copied from this document into the spreadsheet.

1. Begin, as in the previous activity, by generating all of the size factor values. Open a spreadsheet and insert the label "size factor (s)" in cell A1. Type "=0.01" in cell A2, then "=A2+0.01" in cell A3. Copy cell A3 down until you reach the value 1.47, cell A148. This is the maximum size factor that you should have found in Question 5 above. But perhaps the universe can shrink.
2. In cell A149, that follows 1.47, insert "=A148–0.01," then copy it down to cell A294, where the value should be 0.01.
3. Label cell B1 as "size factor change rate," then type "=((3.1/A2)–2.1)^(–0.5)" in cell B2. Copy this cell to the end of the A column values: double click on the cross in the lower right corner of cell B2.
4. Label cell C1 as "years after Big Bang," then insert "=14*0.01*(B2+B3)/2" in cell C3, and "=C3+14*0.01*(B3+B4)/2" in cell C4. Copy this cell to the end of the B column values (use the shortcut).
5. To generate size factor values corresponding to "years after Big Bang," type the label "av. size factor" in cell D1. Then insert "=(A2+A3)/2" in cell D3. Copy this down to the end of the C column.
6. Select columns C and D to make a smooth lined scatter plot, then label the horizontal axis "Years after Big Bang (billions)" and the vertical axis as "size factor." Does the growth of this universe agree with your prediction in Question 7 above?
   $$\underline{\hspace{4cm}}$$

Question 8. According to your graph, what is the current ($s = 1$) age of this particular universe?

Age of the $f_{matter} = 3.1$ universe = _____

Question 9. How does the age of this universe compare to that of the empty universe at $s = 1$?

_____
_____
_____

Question 10. According to the graph, what is the maximum size of this universe relative to its size today? And in how many years after the Big Bang does the universe reach its maximum size?

Maximum relative size = _____

Years to reach maximum relative size_____

Because this universe has a maximum size, it is call "closed," and this is a consequence of the positive curvature ($f_0 - 1 = 2.1 > 0$). Unlike the empty universe, the matter-filled, positive curvature universe (two lines, initially parallel, eventually converge) does not expand indefinitely.

Question 11. According to the graph, how long after the Big Bang does the universe shrink to its initial size?

Years to shrink to 0 size factor = _____

Question 12. If f_{matter} were smaller in a positive curvature universe, would the universe take more years or fewer years to grow and shrink? Why? (Recall that this universe shrinks for the same reason that a ball falls back to earth.)

_____
_____
_____
_____

Dark Energy-Dominated, Flat Universe

As mentioned in the introduction, the actual universe where we live contains microwave radiation from the Big Bang (CMB), ordinary matter (mostly protons, neutrons, and electrons), dark matter (source unknown), and dark energy (source unknown). Cosmologists have made several observations to confirm that these various forms of matter and energy do exist because of their gravitational effects. So we will see how including all of the various forms of matter and energy in the FRW equation affects the size factor of the universe.

Cosmologists have discovered that adding all of the energy fractions gives a value that is very close to one: $f_0 = f_{rad} + f_{matter} + f_{de} = 1$. This is evidence that the universe is flat; two parallel lines always remain parallel. Cosmologists have also determined that f_{rad} is much smaller than f_{matter} and f_{de}. This means that the FRW equation (6) becomes

$$\frac{v^2}{H_0^2} = \frac{f_{\text{matter}}}{s} + f_{\text{de}}s^2 \,.$$

(12)

You will explore two versions of the flat universe, one where there is only matter ($f_{\text{matter}} = 1$, $f_{\text{de}} = 0$), and one where there is matter and dark energy ($f_{\text{matter}} = 0.3, f_{\text{de}} = 0.7$). Cosmologists have determined that the latter f-values describe our current universe.

Equation (12) can be rewritten in a way that allows a prediction of the flat universe's growth:

$$\frac{v^2}{H_0^2} + \left(-\frac{f_{\text{matter}}}{s} - f_{\text{de}}s^2\right) = 0\,.$$

(13)

Question 13. When compared to equation (1) and Question 3 for the ball with zero total energy, how do you expect this universe to behave?

Directions

Remember to choose Edit->Paste Special->Text when pasting formulas copied from this document into the spreadsheet. Both universes will be plotted on the same graph so that they can be compared, so the directions are more complicated than before.

1. First, we'll generate all of the size factor values: open a spreadsheet and insert the label "size factor (s)" in cell A1. Type "=0.02" in cell A2, then "=A2+0.02" in cell A3. Copy cell A3 down until you reach the value 2.8 in cell A141.
2. Label cell B1 "m-only s change rate," then insert "=((1/A2)+0*(A2^2))^(–0.5)" into cell B2. Copy all the way down by double-clicking on the cross in the lower right corner of the cell.
3. Label cell C1 "de&m s change rate," then insert "=((0.3/A2)+0.7*(A2^2))^(-0.5)" in cell C2. Copy all the way down by double-clicking on the cross in the lower right corner of the cell.
4. Type "m-years after Big Bang" in cell D1, and in cell D3 type "=14*0.02*(B2+B3)/2" then in cell D4 insert "=D3+14*0.02*(B3+B4)/2." Copy all the way down in the usual way.
5. In cell E1, insert "av. size factor" and in cell E3 type "=0.5*(A2+A3)," then copy all the way down. Columns D and E give the points for the matter-only graph.
6. For the dark energy & matter universe, type "de&m years after Big Bang" in cell F1, and in cell F3 type "=14*0.02*(C2+C3)/2" then in cell F4 insert "=F3+14*0.02*(C3+C4)/2." Copy all the way down.
7. Finally, in cell G1, insert "av. size factor" and in cell G3 type "=0.5*(A2+A3)," then copy all the way down. Columns F and G give the points for the dark energy & matter graph.
8. Make a smooth lined scatter plot from columns D and E only, the Matter-only flat universe.
9. While holding the mouse over the new graph, right click (for Mac, ctrl click) and choose "Select Data." In the Name box, type "="Matter-Only" " then click on "Add." Type "="Dark Energy+Matter" " in the Name box. For the x-values box, click on cell F3, type a colon, then replace the last "3" by 141. Clear the y-values box, then click on cell G3, type a colon, then replace the last "3" by 141. Click on OK, and the second graph should be plotted along with the first. Consult with you instructor if you don't get the second graph.

10. Label the horizontal axis "Years after Big Bang (billions)" and the vertical axis as "size factor." Does the growth of this universe agree with your prediction in Question 13 above?

_____

Question 14. What would be the age of the universe (for scale factor = 1) if the flat universe contained only dark matter and ordinary matter?

_____

Question 15. Recall that the scale factor is the size of the universe relative to the size of the universe today. In the matter-only universe, how does the early rate of growth of the universe compare to the later rate of growth of the universe. Hint: consider the slope of the graph.

_____
_____
_____
_____

Question 16. In the Dark Energy+Matter universe, how does the early rate of growth of the universe compare to the later rate of growth of the universe.

_____
_____
_____
_____

You should have noticed that the Dark Energy+Matter universe grows faster than the Matter-only universe at later times. Cosmologists have discovered that this is, in fact, what happens. The larger fraction of Dark Energy compared to Matter (0.7 versus 0.3) causes the universe to undergo a growth spurt.

Question 17. Find the current age of the Dark Energy+Matter universe, where we live.
Age of our universe = _____

Question 18. What are your estimates of the size factor and the age of the actual universe when it begins to grow at a faster rate than the Matter-only universe?
s = _____   age = _____

From several different observations, cosmologists have determined that the universe is 13.7 billion years old.

Although a total energy of zero (with $f_{matter} = 0.3$ and $f_{de} = 0.7$) for our universe will cause it to rapidly increase its expansion rate beginning about 20 billion years after the Big Bang, the reason for the rapid growth is not easily understood from our previous discussion. We must return to the FRW version of Einstein's theory for an explanation. Unlike the case of the ball with zero energy escaping the earth because of adequate escape speed, the dramatic expansion of our universe will occur because the dark energy exerts a *negative* pressure that acts as anti-gravity on the entire universe.