Methods of Period Determination in RV Tauri Stars

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Abstract

We have tested a number of methods to better determine the periodicity of RV Tauri stars. The main methods we examined are: visual inspection, manual phasing, automated phasing, curve fitting, the weighted wavelet Z method, and the Fourier analysis tools with the AAVSO's TS12 program. We have used data from the AAVSO database for several RV Tau stars to test each of the period finding methods. Here we discuss each method and their strengths and weaknesses when applied to RV Tau lightcurves.

Background

This poster details the summer research project of the authors, Joshua Davis and Graham Davis, under the NSF Partnership in Observational and Computational Astronomy (POCA) award at South Carolina State University (SCSU). One major aspect of the POCA project focuses on developing a better understanding of the variable star category of RV Tauri stars. Although the entire RV Tau research project is working to combine photometric and spectroscopic analysis, our portion of the research for the summer of 2008 focused on testing various methods of period determination for the RV Tau visual lightcurves.

RV Tau stars are pulsating yellow super giants whose light curves are characterized by alternating deep and shallow minima. The period from one deep minimum to the next (the 'double' or 'formal' period) ranges from 30 to 150 days. The RV Tau type stars are considered "semi regular" meaning that there is significant periodicity in the light curve but it also includes irregularities. In RV Tau stars, we can see times when the normal pattern of alternating deep and shallow minima breaks down, variability in the formal period, and sudden changes in the average magnitude of the light curve.

Because RV Tau show these types of irregularities determining the period is both important and difficult. We are testing different methods of period determination to help us find the most appropriate method for our RV Tau stars. As a student project, we are also working to build our understanding of the basics of RV Tau stars and period determination methods.

Understanding the Data

Two main types of data were used in our work, visual observations from the AAVSO database and synthetic data. We started by looking at the AAVSO data of several known RV Tau stars. This initial examination of the data was used to familiarize us with the typical lightcurves of these stars and allowed us to select a few light curve regions to focus our investigation.

For the overall study of RV Tau stars at SCSU we are starting with specific stars which have good AAVSO coverage and spectral data. Nine particular ones were selected based on the above criteria; R Sct, Z Uma, SX Her, AC Her, U Mon, RV Tau, SU Gem, TX Per and V Vul. For the summer project we have primarily worked with U Mon, V Vul, Z Uma and SU Gem. The AAVSO database has observations for these stars going back decades and trying to work with the entire lightcurve at once is difficult. By breaking the data into smaller sets with hundreds to thousands of days worth of information, we were able to get a better feel for the variability that we are likely to encounter in RV Tauris and find some regions to use in our testing. We started by identifying with some regions that show the regular deep-shallow minima pattern but need to keep in mind the complications that we will encounter in future analysis. A few data sets that we looked at are included in the figures to the right. Figure 1 and 2 show regions of observational data used in the study that have the typical lightcurve pattern. Figure 3 shows a larger time span for U Mon showing the irregularity in the average magnitude level.

We also worked with synthetic data for the initial testing of each of our period determination methods. The purpose of this was to get a better understanding on how to analyze the data using known signals to check the accuracy of our results. Most of the synthetic data sets had a single sine curve signal, some pure sine curves, some with random noise added, and then some with irregular gaps in the data. We also constructed a few data sets consisting of a secondary sine curve with a period approximately half of the first sine curve. These "two sine" synthetic lightcurves were constructed to more closely resemble the multi-periodic lightcurves seen in RV Tauri systems. We found that the spacing between data points, noise level and presence of a secondary period within the data all affected how we determined the period.

Understanding the Methods

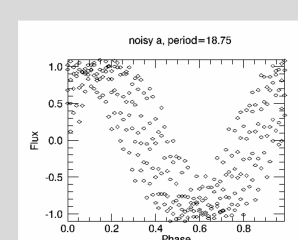
Visual Inspection:

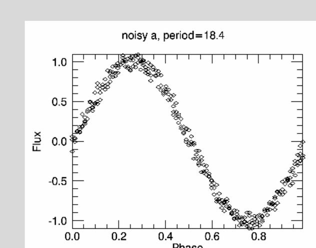
A first approach to determine the period of a light curve is to look at the timing of the minima of the light curve. If the data is fairly complete over at least one cycle, we could count the days from minimum to minimum to get a rough estimate of the period. We can improve the estimate by averaging over several cycles. If the data has enough gaps that we can not determine the time of each minima, we can still estimate the period by taking the total number of days in the light curve and divided it by the total number of cycles.

The visual inspection of the light curve is a very simple method that can be applied to light curves where the overall cycles are easily identified and helped us to really understand the concept of periodicity. This method has some major limitations however. If the data is sparse or has large gaps, it becomes much more difficult to confirm the number of cycles present and the timing of each minima. At best we should consider the results we get from this method as a starting place for the other methods.

Manual Phasing:

As a second approach to determine the period of a light curve, we can use manual phasing of the data to refine the estimated period of the data. By phasing, we mean that the time axis of the light curve data is transformed into phase by dividing the time by the estimated period and keeping only the fractional decimal. The phase then runs from 0-1 and marks the relative position of each data point with respect to the cycle. We then look at the spread of the phased light curve to refine the estimated period. As we move toward the correct period, the points on the phased light curve become less spread out and the shape of the cycle becomes more evident. Below we show two manually phased light curves of one our synthetic datasets. The first shows the estimated period we obtained from a visual inspection of the light curve and the second shows a refinement of the period. It can easily be seen on these graphs that the correct period must be very close to 18.4 days since this graph show a very narrow spread of the data points.





Manual phasing of the data is simple and very easy to understand and provides a good visual confirmation of the estimated period. Depending on the amount of noise present in the data you can get a fairly precise value for the period but it can be very time consuming. In addition you really need a good initial estimate of the period to start. Its accuracy and usefulness is also limited by the regularity of the data. If the data has irregularities in it, such as sudden changes in the average brightness or more than one strong period in the light curve, the phased light curves will become less recognizable. Since the RV Tau observational data is known to have these issues as well as a significant amount of natural dispersion, manual phasing of the light curves will not be a practical method to determine the period. We may want to use the manual phasing method as a check after we have used another method to find a possible period in the data.

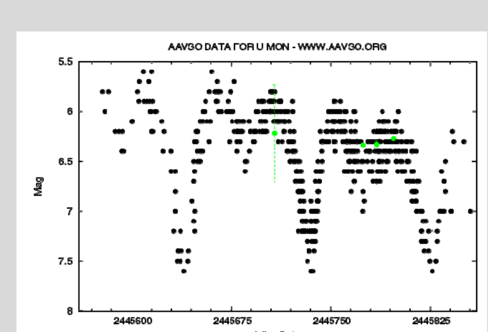


Figure 1

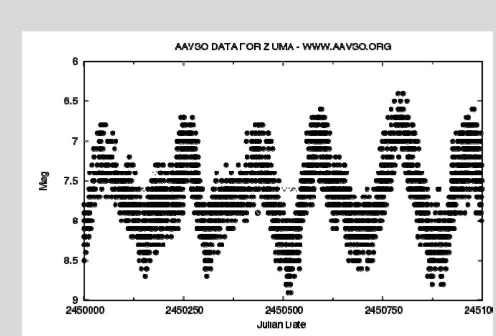


Figure 2

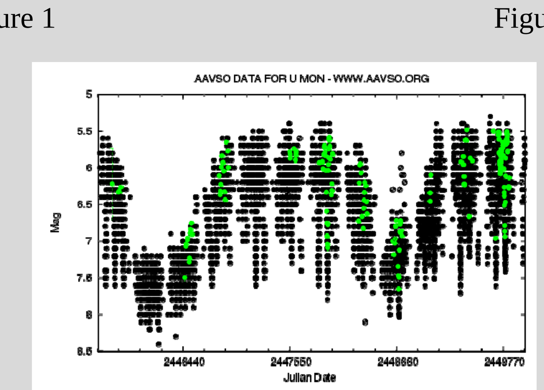


Figure 3

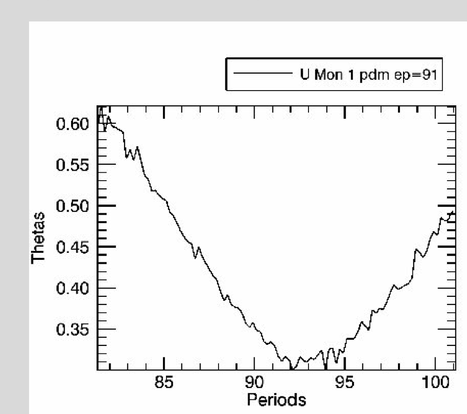
Automated Phasing:

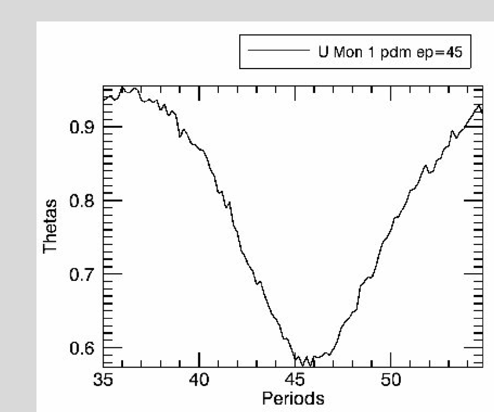
Two of the limitations of the manual phasing method can be overcome by automating the phasing process. In these automated methods, the data is phased on an estimated period and then used to calculate a mathematical estimate of how good each period estimate is at creating a smooth lightcurve. The automated methods then go on to test a range on possible periods and determine the best period within the test range. We used IDL to create our two automated phasing programs, string and PDM.

The string program is based on the methods and equations provided by Dworetsky 1983. After we created a phased light curve and sorted all points by phase, the program calculates the "string lengths" for each period by calculating the cumulative distance connecting each data point in turn. When the correct period is used to phase the light curve, the string length should be at a minimum. As you move further away from the correct period the string length increases.

The PDM method, which stands for Phase Dispersion Minimization, is based on the methods and equations provided by Stellingwerf 1978 and breaks the phased lightcurve into bins and tries to minimize the spread of the data points. When the correct period is used, the phase dispersion should be at a minimum. As you move further away from the correct period the dispersion becomes greater.

We find that the string method is best suited to situations where there is a small number of data points. While this may work to look for periodic signals within the spectroscopic data, it is not going to work for the much larger photometric dataset. The PDM method, on the other hand, works fairly well both the synthetic and AAVSO data. When applied to a time span of 1000-2000 days worth of AAVSO data, the PDM method found a period consistent with the published periods for each system we looked at. The figures shown below are the PDM results for one section of U Mon data centered around the known formal period and half period. The precision is somewhat limited due to the inherent dispersion within the AAVSO data and the somewhat irregular nature of the RV Tau lightcurve and this precision is probably not sufficient to really study variations in the periodicity. We can probably use the PDM method to help quantify the level of irregularities in the lightcurve over longer spans of time.





Curve fitting:

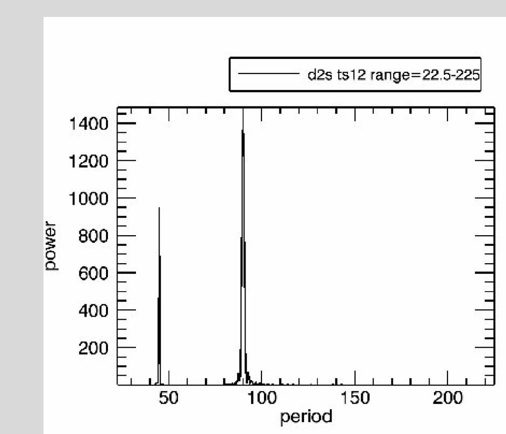
Another method to determine the period of a lightcurve is to fit the light curve signal with a periodic function and examine the residuals between the observed data and the estimated function. When the residuals are smallest, you have the best fit curve and can extract the period as well as the amplitude and zero phase point. While we were testing the manual phasing method on the synthetic datasets, we also tested a basic manual curve fitting. In this case, we assumed that we knew the amplitude and zero phase point and used the period that we were testing for the manual phasing. By examining the residuals of the curve fit, we could more easily see when our period estimate from the manual phasing we correct.

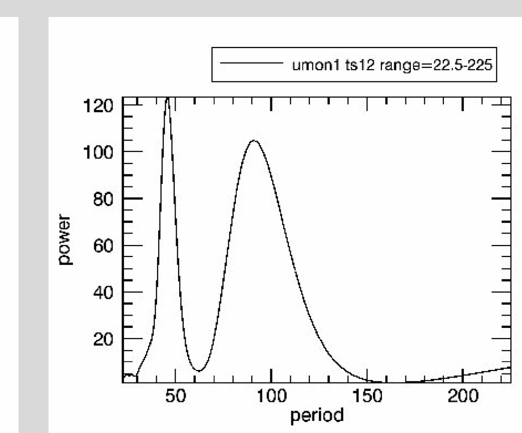
As we moved into the automated phasing method for determining the period, we could also have moved to an automated curve fitting routine. In that case we would have had to vary the period, amplitude, and zero phase point at the same time. In addition, for the "two sine" synthetic data and the observational data from the AAVSO, a single sine curve fit would be insufficient to model the data. Instead of pursuing curve fitting as an independent method of determining the period, we feel that it can best be used to follow up on the results found from other methods, by modeling the lightcurve with the estimated period and then examining the residuals.

TS12

TS12 is a time series analysis program that reads in a data set and can perform various functions including Fourier analysis. We obtained the TS12 program from the AAVSO website. Fourier analysis is based on the concept of using a Fourier series to model observed data as a linear combination of sines and cosines. Fourier transforms are complicated mathematical operations that take an observed dataset and determine the strength of each frequency in the data (frequency and period are inversely related). TS12 prints out a plot of a range of period estimates and the strength of each period. The stronger a power is, the closer it is to the correct period. A more detailed description of the TS12 software is given in Foster 1994.

We were able to use the TS12 software on a few of our datasets. The program found both the formal and half period in both the "two sine" synthetic data and the U Mon observational data as shown in the figures below. We did have some troubles getting some of our synthetic data sets to run in the software and had troubles with errors on some of the observational datasets. With additional time we should be able to get the software running correctly on all of our data. We have not yet had a chance to fully explore all of the features in the TS12 software, but early indications are promising for its future use. In particular we need to see how the software will react to sections of RV Tau data where there are clear irregularities.

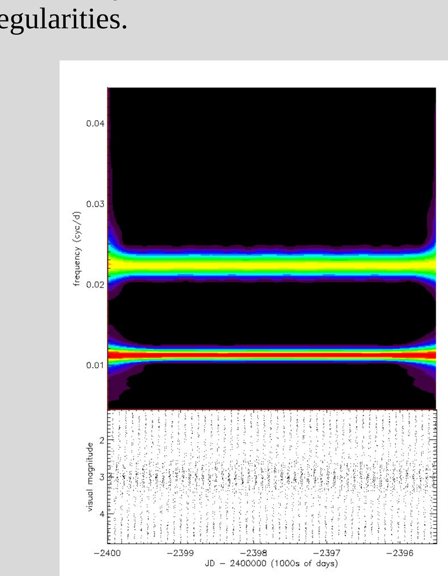


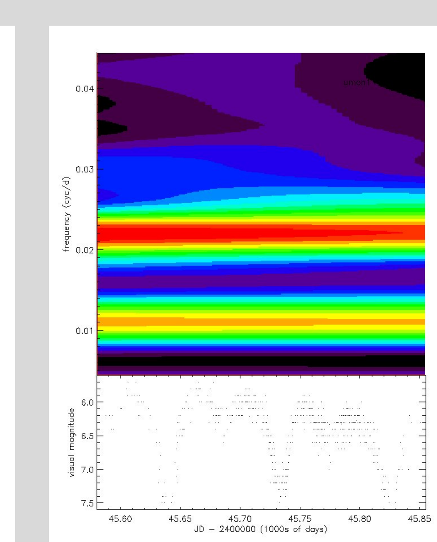


WWZ

Another method that was used is called the weighted wavelet Z-transform method (WWZ). We obtained the WWZ program from the AAVSO website. WWZ is based on Fourier analysis but focuses on small windows of time and fits the data with wavelets instead of full sine and cosine waves. This method scans in a data set and produces a graph with multiple frequencies and the strength of each frequency as a function of time. This is produced in the form of a false color graph where inaccurate frequencies are dark and more accurate frequencies are represented by red or white.

Examples of the output of the WWZ program for one of our "two sine" synthetic datasets as well as the results for U Mon are given in the figures below. Again, the two dominate periods are clearly visible in both cases. The fact the WWZ can track changes in the behavior of the light curve over time has the possibility of being very useful in the analysis the irregularity of RV Tauri stars. Again, additional time is needed to complete a more careful analysis of additional datasets and larger observational datasets that include clear irregularities.





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- Analysis was performed using the computer program IDL.
- Analysis was performed using the computer program TS, developed by the American Association of Variable Star Observers.
- Wavelet analysis was performed using the computer program WWZ, developed by the American Association of Variable Star Observers.
- We acknowledge the guidance of our research mentor, Dr. Jennifer Cash.

References:

- Stellingwerf R, 1978, "Period Determination Using Phase Dispersion Minimization", ApJ, 224, 953
- Dworetsky M, 1983, "A period-finding method for sparse randomly spaced observations or 'How long is a piece of string?'", MNRAS, 203, 917
- Foster G, 1994, "The Cleanest Fourier Spectrum", AJ, 109, 1889
- Foster G, 1995, "Wavelets for Period Analysis of Unevenly Sampled Time Series", AJ, 112, 1709
- Percy, John. Understanding Variable Stars. New York: Cambridge University Press, 2007.