Fractal and Wavelet Image Compression of Astronomical Images

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Overview of Image Compression

The advancement of digital technology has led to a sweeping use of digital imagery. With this industry rapidly expanding, the applications of electronic data processing, the need for mass information storage and faster communication links continues to grow. Consequently there has been and increased interest in compression of image data. Two areas in mathematics that have contributed to enhancements of image compression include fractals and wavelets.
Advantages of Image Compression

- Increases the rate of data transmission
- Reduces the amount of memory used from the computer’s hard drive required to store an image
- Also reduces the amount of memory used to send and retrieve the image.
Without the Application of Compression...

<table>
<thead>
<tr>
<th>Data</th>
<th>Dimension</th>
<th>Uncompressed Size</th>
<th>Transmission Time (28.8K Modem)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gray Scale Image</td>
<td>512 x 512</td>
<td>262 KB</td>
<td>1 min 13 sec</td>
</tr>
<tr>
<td>Color Image</td>
<td>512 x 512</td>
<td>786 KB</td>
<td>3 min 39 sec</td>
</tr>
<tr>
<td>Video</td>
<td>640 x 480</td>
<td>1.66 GB</td>
<td>5 days 8 h</td>
</tr>
</tbody>
</table>
How Does Compression Work?

A common characteristic of most images is that the neighboring pixels are correlated and therefore contain redundant information. By removing a combination of the redundant information along with “irrelevant” data (data that will be unnoticed by the human eye), the size of an image file can be significantly reduced.
Types of Compression

- **Lossless Compression**
  
  - With lossless compression techniques, the decompressed image is identical to the original image.
  
  (1\textsuperscript{st} level Haar wavelet, threshold: $1/2^{16}$)
Lossy Compression

- An image reconstructed following lossy compression contains degradation relative to the original. However, lossy compression methods are capable of achieving much higher compression.

(1st level Daub4 wavelet, threshold: $1/2^2$)
Wavelet Compression

A computer stores a 256 x 256 grayscale image as a matrix with each element ranging from 0 (black) to a positive whole number (white). By averaging and differencing the elements of this matrix, a new matrix is formed. Wavelets can be used in the process of compressing 1 dimensional and 2 dimensional signals.
What are Wavelets?

A wavelet is a mathematical function useful in digital signal processing and image compression. In image processing, wavelets make it possible to de-noise polluted images (Noise is unwanted electrical or electromagnetic energy that degrades the quality of signals and data).
Types of Wavelets

- **Haar Wavelets**
  - Haar wavelets are the simplest forms of wavelets. The resulting signal, after the application of a Haar transform, will not be smooth.

- **Daubechies wavelets**
  - More complex than the fundamental Haar wavelets, however they produce a “smoother” image.
The Haar Wavelet Transform of a 1-dimensional signal

Let

\[ \vec{f} = (f_1, f_2, \ldots, f_{2N}) \]

be a 1-dimensional signal

The Haar Wavelet Transform decomposes the signal \( \vec{f} \) into two parts
The Haar transform

★ Trend (average): \( \vec{a} = (a_1, a_2 \ldots, a_m) \)
\[
a_m = \frac{f_{2m-1} + f_{2m}}{\sqrt{2}}
\]

★ Fluctuation (difference): \( \vec{d} = (d_1, d_2 \ldots, d_m) \)
\[
d_m = \frac{f_{2m-1} - f_{2m}}{\sqrt{2}}
\]
The Haar Wavelet Transform of a 2-dimensional signal

- An image \( \tilde{f} \) (2-dimensional signal) is a rectangular M x N matrix

\[
\tilde{f} = \begin{pmatrix}
\tilde{f}_{1,1} & \tilde{f}_{1,2} & \cdots & \tilde{f}_{k,N} \\
\tilde{f}_{2,1} & \tilde{f}_{2,2} & \cdots & \tilde{f}_{k,N} \\
\tilde{f}_{3,1} & \tilde{f}_{3,2} & \cdots & \tilde{f}_{k,N} \\
\vdots & \vdots & \ddots & \vdots \\
\tilde{f}_{k,N} & \cdots & \cdots & \tilde{f}_{k,N}
\end{pmatrix}
\]
The Haar Wavelet Transform of a 2-dimensional signal (continued)

2-dimensional Haar Wavelet transform consists of two steps:

- Perform 1-dimensional Haar transform to each row of the image $\tilde{f}$.
- Perform 1-dimensional Haar transform to each column of the matrix obtained in the first step.
Advantages of Applying Wavelet Compression

In some cases, a wavelet-compressed image can be as small as about 25 percent the size of a similar-quality image using the more familiar JPEG method. Thus, for example, a photograph that requires 200 KB and takes a minute to download in JPEG format might require only 50 KB and take 15 seconds to download in wavelet-compressed format.
Fractal Compression

There are two major benefits of changing images to fractal data. The first benefit is that the size of memory used to store fractal codes is tremendously smaller than the amount of memory used to store the original bitmap information. The second benefit of converting an image into fractal data is that since the data is mathematical, the image can easily be scaled up or down a size (zooming) without disrupting the detail of the image.
Fractals (continued)

- A fractal is a structure that is made up of similar forms and patterns that occur in many different sizes.
- These patterns appear nearly identical in form at any size and occur naturally in all things.
- The key property that characterizes fractals is self-similarity.
Fractal image compression is based on the concept of an Iterated Function System (IFS).

The final output after an IFS is implemented is called the attractor.

The attractor below is the Sierpinski triangle.

It is a fractal.
Softwares Used in Compressing Astronomical Images

- FAWAV (Fourier Analysis Wavelet Analysis)
- H-compression
- AIP (Astronomical Image Processing)
- SpiePress
Can perform Fourier and wavelet analysis on digital 1 and 2 dimensional signals.

FAWAV allows a user to perform several types of Fourier and wavelet analysis.

In FAWAV, once a signal is graphed, you can determine the statistics and energy of the signal.
Statistics

- Maximum and minimum values of the signal
- Mean
- Variance
- Standard deviation
- Integral
- Derivative
- Scalar product
- Number of samples
- Energy
**H-compress**

Image compression package written to compress Telescope Science Institute (STSci) digitized sky survey images.

- Well suited for the compression of astronomical images
- Can be used for either lossless or lossy compression
- Based on the H-transform of the image
  - H-transform is a 2 dimensional generalization of the Haar transform
- H-compress is operated on the UNIX system
- WS_FTP was used to view the decompressed file
Original and H-compressed Image

- Compression factor of 8
Original Image and Statistics
Compressed Image and Statistics
AIP

Astronomical Image Processing System is an image processing software package. This software can be used to:

- Calibrate and construct the display and analysis of astronomical images
- View images that are in FITS (Flexible Image Transport System) format.
AIP (continued)

- AIP was designed to run, with minimal modifications, in a wide variety of computing environments.
- AIP is now well exploited in the astronomical community. It is currently in active use for astronomical research at approximately 250 sites worldwide.
**SpiePress**

This software allows us to explore and experiment with fractal and wavelet image processing. There are three complete windows compatible software:

- **IFS System**
  - Allows users to create their own fractal images using iterated function system

- **Img System**
  - Compresses images using fractal techniques and displays decoded images

- **WAV System**
  - Performs similar functions on images using wavelet techniques and also displays the image’s wavelet transform
SpiePress (using Img System)

- Decoding results using 16 x 16 square
- Error threshold: 0.005
- 10 Iterations
SpiePress (using WAV System)

Compression percent 50%
Original Image with Statistics
Image after application of Fractal Compression
Image using Wavelet Compression

* Compression percent: 50%
References

- M. F. Barnsley, *Fractal Image Compression*, Notice of AMS, Vol. 43, Number 6, 1996
- K. Adzievski, class notes